Directed polymers with random interaction: An exactly solvable case

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We propose a model for two $(d+1)$-dimensional directed polymers subjected to a mutual δ-function interaction with a random coupling constant and present an exact renormalization-group study for this system. The exact β function, evaluated through an ϵ $(=1-d)$ expansion for second and third moments of the partition function, exhibits the marginal relevance of the disorder at $d=1$ and the presence of a phase transition from a weak- to strong-disorder regime for $d>1$. The length-scale exponent for the critical point is $\nu=(2|\epsilon|)^{-1}$. We give details of the renormalization. We show that higher moments do not require any new interaction, and hence the β function remains the same for all moments. The method is extended to multicritical systems involving an m-chain interaction. The corresponding disorder-induced phase transition for $d>d_m=1/(m-1)$ has the critical exponent $\nu_m=[2d(m-1)-2]^{-1}$. For both the cases, an essential singularity appears for the length scale right at the upper critical dimension d_m. We also discuss the strange behavior of an annealed system with more than two chains with pairwise random interactions among each other.

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I. INTRODUCTION

Attempts to study the effects of randomness, especially if one requires averages of thermodynamic quantities, have led to many new techniques, concepts, and, probably, controversies [1]. In order to get a clear idea about random systems, in recent years, a directed polymer (DP) in a random medium seems to have emerged as the consensus candidate for the "simplest" random model [2–6]. Here we propose a still simpler problem of DP's with random interaction that can be solved using an exact field-theoretic renormalization-group (RG) approach [7]. This, we believe, is highly significant since RG is the general framework to study and to understand, through the fixed-point spectrum, the universal aspects of any model.

Directed polymers in $(d+1)$ dimensions are random walks directed along a particular direction, say z, with fluctuations in the transverse d-dimensional space. DP's are of considerable interest and have attracted much attention as a simple statistical-mechanical model because of its relevance and applicability in unifying a wide variety of seemingly disparate systems. These include the flux lattice-melting problem in high-T_c superconductors [8], commensurate-incommensurate transitions [9], wetting transition [10, 11], vertex models [12], nematic polymers [13], biomembrane phase transitions [14], interface growth [15], etc. Many problems of conventional polymers [self-avoiding walks (SAW's)] such as collapse, adsorption, etc. have exactly solvable counterpart in DP's [16]. The RG analysis of a pure system of interacting DP's gives enough insight through the evaluation of the exact β function to all orders in perturbation series [17, 12, 18]. These systems of DP's with pure short-range interactions are almost completely solved and have led, for example, to several exact results for vertex models [12, 19].

There are many efforts and activities in the field of polymers with random interaction [20] or in random media [21]. The analogous DP problems are expected to be simpler. For example, a DP in a random medium, which through a nonlinear mapping describes many aspects of interface growth, has been studied up to one loop in the momentum-shell technique [15]. There also were attempts for solving the many-chain system in a random medium in the context of high-T_c superconductivity [22]. It is the directedness that helps in setting up the DP problem, both analytically and numerically, as oppposed to the SAW problem in random media. Several results for the DP problem are known in general, though exact or rigorous results are rather few [3]. Apart from these random-media problems, the other category of problems involves polymers with random interactions in the context of, say, disordered heteropolymers [20]. Here again, a DP with random interaction turns out to be simpler than ordinary polymers [23].

Our model [7] has a similarity to the second category of problems. It deals with a random *mutual* interaction among the chains with the randomness in the coupling constant of the interaction. The randomness is only along the length of the chains and does not depend on the transverse d-dimensional coordinates. The specific characteristic of the randomness as well as the directed nature of the polymers enable us to solve the model exactly. Furthermore, we show that this model, in spite of its simplicity, captures many of the essential features such as marginal relevance, existence of a disorder-induced phase transition, etc. as known, e.g., for the interface-growth problem, DP and SAW in random environments, etc. [4, 21]. The two-dimensional wetting phenomenon is also analogous to our proposed system [11, 24], although our model (and the solution) is for general d.

We define the model in the next section, and to put things in the proper context, the aim and the outline of the paper are given there.

II. MODEL

From the definition of DP's, it follows that a projection of a DP in the transverse d-dimensional space is an ordinary polymer with z representing the contour variable which is equivalent to the step length in a discrete case. In the path-integral formulation the dimensionless Hamiltonian for two such DP's, each of length N, interacting through a random mutual short-range interaction can be written as [7]

$$H = \frac{1}{2}\int_0^N dz \left[\left(\frac{\partial \mathbf{r}_1(z)}{\partial z}\right)^2 + \left(\frac{\partial \mathbf{r}_2(z)}{\partial z}\right)^2\right] + \int_0^N dz\, v_0\,[1+b(z)]\, V(\mathbf{r}_{12}(z)), \tag{2.1}$$

where $\mathbf{r}_i(z)$ is the d-dimensional position vector of a point of chain i at a contour length z, and $\mathbf{r}_{12}(z) = \mathbf{r}_1(z) - \mathbf{r}_2(z)$. The first term that comes from the chain connectivity is the entropic contribution, and it corresponds to free chains [25]. The second term is the two chain interaction at the same chain length through a short-range potential $V(\mathbf{r})$. We introduce the randomness through the coupling constant. It has a pure part v_0 and a random part $v_0 b(z)$, which varies only with z (the length along the chain). It is chosen in this way so that $b(z)$ is dimensionless. At this stage, for generality, we keep V as a short-range potential. Later on, specific calculations would be done with a δ-function potential. Also, starting with a short-range potential has certain mathematical advantages such as avoiding powers of distributions, as we will see below. One can also think of this problem as a nonrelativistic quantum problem of particles with a time (z) -dependent interaction potential—a description we do not find very illuminating.

One of the simplest but nontrivial choices for the distribution of the randomness is a Gaussian one:

$$P(b(z)) = (2\pi\Delta)^{-1/2}\ \exp[-b(z)^2/(2\Delta)], \tag{2.2a}$$

$$\langle b(z)\rangle = 0, \text{ and } \langle b(z_1)b(z_2)\rangle\ = \Delta\ \delta(z_1 - z_2). \tag{2.2b}$$

Here the randomness is uncorrelated in nature and is described by the variance Δ. Choosing a zero mean for $b(z)$ is not a restriction because any nonzero $\langle b(z)\rangle$ could be made to vanish by adding it to (and thereby redefining) the pure part.

So far we have discussed only two-body interactions. For DP's it is known that even pure many-body interactions, representing special multicritical points, can also be handled exactly [26, 19, 18]. It is possible to study the disordered versions of these multicritical systems. The Hamiltonian for the mth-order multicritical point, involving only an m-body δ-function interaction, is

$$H_m = \frac{1}{2}\int_0^N dz \sum_{i=1}^{m}\left(\frac{\partial \mathbf{r}_i(z)}{\partial z}\right)^2 + \int_0^N dz\, v_m\,[1+b(z)] \prod_{i=1}^{m-1}\delta(\mathbf{r}_{i\ i+1}(z)), \tag{2.3}$$

where, as before, $b(z)$ is the random part. We come back to this multicritical situation in Sec. VI. It is also possible to define more general systems by putting the lower-order interactions in Eq. (2.3) with independent random coupling constants. Such a Hamiltonian can, in principle, describe the approach to the multicritical points. However, such complicated cases are not discussed here.

One possibility of getting a random interaction for, say, the two-chain case is to take "charged" DP's, with random charges $q_i(z)$ for the ith chain, $v_0 b(z) = q_1(z)q_2(z)$, and interactions of charges only at the same z. If the charges are in thermal equilibrium with the polymers, a simple quadratic Hamiltonian for the charges can be taken to be proportional to $\int b(z)^2\, dz$ and it is to be added to the Hamiltonian of Eq. (2.1). The partition function one gets from this full Hamiltonian is really equivalent to $\langle Z\rangle$ (annealed case) evaluated with the Hamiltonian of Eq. (2.1) but averaged over the distribution of Eq. (2.2a). The more complex situation is the quenched average which requires, e.g., the average of the free energy.

One of the standard approaches for random systems is to proceed through the evaluation of the quenched free energy using the replica trick [1]

$$\langle \ln Z\rangle = \lim_{n\to\infty}\frac{\langle Z^n\rangle - 1}{n},$$

which requires the evaluation of $\langle Z^n\rangle$. The importance of these moments can be realized through the following expansion:

$$\langle \ln Z\rangle = \ln\langle Z\rangle + \sum_{n=2}^{\infty} n^{-1}\langle [Z/\langle Z\rangle - 1]^n\rangle.$$

Such an expansion makes sense if and only if the various cumulants of the partition function, with respect to the disorder distribution, do not grow too rapidly with n. In such a case there would not be much qualitative difference between the quenched and annealed cases. This, in turn, suggests that, to look into the possible differences, one can study the various moments of the partition function. In addition, the moments can be looked upon as the charateristic function for the probability distribution of $\ln Z$ [27]. They are therefore of interest in themselves [28]. This is the approach we take in this paper. Our analysis is not yet enough for the analytic continuation in n to the $0 \le n \le 1$ regime as one would need for the free energy. This is not a deterrence since important information can be gathered even from the integral moments.

Instead of evaluating the quenched free energy, we systematically study the behavior of $\langle Z\rangle$, the second and the third cumulants. The first moment describes the behavior of an annealed system while the higher cumulants would show the nature of fluctuations. For each case, we do the averaging exactly before the configuration sum (or "path integrals") to define an effective Hamiltonian for that particular cumulant. This "pure" effective Hamiltonian is then treated by perturbative renormalization. The effect of disorder is felt through the generation of new terms in the effective Hamiltonian. The RG analysis helps in examining the flow of these terms as the length scale is increased, thereby showing the marginal

relevance or irrelevance of the disorder.

In Sec. III, $\langle Z \rangle$ is discussed, while $\langle Z^2 \rangle$ and $\langle Z^3 \rangle$ are presented in Secs. IV and V. Though the derivation of effective Hamiltonian precedes perturbative analysis in Secs. III–V, it is instructive to start with the original Hamiltonian, do the perturbation analysis, and then do a term by term disorder averaging. Such a procedure not only shows how the new terms are generated but also acts as a cross-check. This is discussed in Appendix A. Appendix B discusses many of the details needed in Sec. IV. The random multicritical case is discussed in Sec. VI. The annealed case of three and four chains is discussed in Sec. VII. A discussion and summary are presented in Sec. VIII.

III. $\langle Z \rangle$

We show in this section that the annealed case can be reduced to a pure problem. We add that this reduction is special for two chains. Had we started with more than two chains, say three or four, with the same random pairwise interaction as in Eq. (2.1), the annealed case would be completely different from the corresponding pure case and we would have a much richer structure. A particular case is discussed in Sec. VII.

The partition function, in the continuum approach, for a system of two chains, given by the Hamiltonian of Eq. (2.1), is

$$Z = \int Dr_1\ Dr_2\ \exp(-H),$$

where $\int Dr_1 Dr_2$ stands for the sum over all configurations of the two chains. A straightforward averaging of Z using the probability distribution of Eq. (2.2a) defines an effective Hamiltonian $\mathcal{H}_{\text{eff}}$ such that

$$\langle Z \rangle = \int Dr_1\ Dr_2\ \exp(-\mathcal{H}_{\text{eff}}), \tag{3.1}$$

and it is given by

$$\mathcal{H}_{\text{eff}} = \frac{1}{2}\int_0^N dz \sum_{i=1}^{2}\left(\frac{\partial \mathbf{r}_i(z)}{\partial z}\right)^2 + v_0\int_0^N dz\ V(\mathbf{r}_{12}(z)) - \frac{v_0^2\Delta}{2}\int_0^N dz\ V^2(\mathbf{r}_{12}(z)). \tag{3.2}$$

It appears from the above expression of the effective Hamiltonian that an attraction is generated between the two chains. We find it instructive to follow another approach of perturbation expansion of the interaction term starting with the original Hamiltonian (2.1). This helps us in visualizing the origin of the disorder-induced attraction. This is done in Appendix A.

Now, since any short-range potential under renormalization maps onto a δ-function potential, we can take the "minimal" effective Hamiltonian for $\langle Z \rangle$ as

$$\mathcal{H}_2 = \frac{1}{2}\int_0^N dz \left[\left(\frac{\partial \mathbf{r}_1(z)}{\partial z}\right)^2 + \left(\frac{\partial \mathbf{r}_2(z)}{\partial z}\right)^2\right] + \bar{v}_0\int_0^N dz\ \delta(\mathbf{r}_{12}(z)), \tag{3.3}$$

where $\bar{v}_0$ is the reduced coupling constant which takes care of the attraction described earlier. We believe that the large-length-scale properties as described by Eq. (3.3) are the same as those of Eq. (3.2). If necessary, we can restrict the strength of the disorder so that $\bar{v}_0$, which represents the effective coupling between the two chains, is positive (i.e., repulsive interaction). Now the problem reduces to a relatively simple situation where the two chains interact with a pure δ-function interaction with a reduced coupling constant $\bar{v}_0$. The solution of this pure problem is known and is used below [17, 12].

For the sake of completeness we quote the relevant results from from Ref. [17]. The perturbation series for the connected part of the annealed partition function $\langle Z \rangle_c$ to all orders in $\bar{v}_0$ is

$$\langle Z \rangle_c = N\mathcal{V}\bar{v}_0\left[1 + \sum_{n=1}^{\infty}(-1)^n \frac{\bar{v}_0^n}{(4\pi)^{nd/2}} N^{n\epsilon'} \frac{\Gamma^n(\epsilon')}{\Gamma(2+n\epsilon')}\right], \tag{3.4}$$

where $\mathcal{V}$ is the d-dimensional transverse volume and $\Gamma(\ )$ is the standard gamma function. The exact β function for the renormalized coupling constant u (with $u_0 = \bar{v}_0 L^{2-d}$ as the bare dimensionless coupling constant) is

$$\beta(u) \equiv L\frac{\partial u}{\partial L} = 2\epsilon' u\left(1 - \frac{u}{4\pi\epsilon'}\right). \tag{3.5}$$

Note here that $2\epsilon' = (2-d)$ replaces ϵ of Ref. [17] to avoid later conflict of notation.

The flow diagram for the dimensionless coupling constant u is shown in Fig. 1. The fact that for $d < 2$ any small attractive interaction is able to form a bound state is reflected by the flow to the nonperturbative regime for any negative u. The repulsive or the positive u region is dominated by the stable fixed point u^* $(= 4\pi\epsilon')$. For $d > 2$ there exists a nontrivial unstable fixed point $u = u^*$ which seperates the bound and the unbound states for the two polymers. In short, the unstable fixed point represents the critical point for the binding-unbinding transition. The exponents are known and can be found in

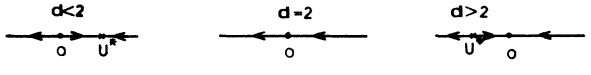

FIG. 1. Flow diagrams for coupling constant u in different dimensions. u^* $(= 4\pi\epsilon')$ represents the nontrivial fixed point.

Ref. [29]. For example, for $1 \leq d < 4$, the length scale exponent is $1/|\epsilon'|$ [29].

IV. $\langle Z^2 \rangle$

The evaluation of $\langle Z^2 \rangle$ closely parallels that of the preceding section. However, unlike the $\langle Z \rangle$ case, new terms are generated here in the effective Hamiltonian. An RG analysis is done to get a detailed account of the effects of these new terms.

A. Effective Hamiltonian

The averaging for $\langle Z^2 \rangle$ with the Hamiltonian in Eq. (2.1) needs completion of the perfect square associated with $b(z)$. As in the replica analysis [1] where one needs n replicas ("copies") of the original system in evaluating $\langle Z^n \rangle$, we require four chains for $\langle Z^2 \rangle$, a pair {3,4} as a replica of the original pair of chains {1,2}. Therefore, we write, restricting ourselves to δ-function potentials,

$$\langle Z^2 \rangle = \int \prod_{i=1}^{4} Dr_i \quad \exp(-\mathcal{H}_{2,2}), \tag{4.1a}$$

where

$$\mathcal{H}_{2,2} = H_0 + H_1 + H_2, \tag{4.1b}$$

with

$$H_0 = \frac{1}{2} \int_0^N dz \sum_{i=1}^{4} \left(\frac{\partial \mathbf{r}_i(z)}{\partial z} \right)^2 \tag{4.1c}$$

denoting the four-chain free part and

$$H_1 = \bar{v}_0 \int_0^N dz \; [\delta(\mathbf{r}_{12}(z)) + \delta(\mathbf{r}_{34}(z))] \tag{4.1d}$$

and

$$H_2 = -\bar{r}_0 \int_0^N dz \; \delta(\mathbf{r}_{12}(z)) \; \delta(\mathbf{r}_{34}(z)) \tag{4.1e}$$

representing the interactions among the chains, with $\bar{r}_0 = v_0^2 \Delta$.

In Eq. (4.1b), H_1 denotes the repulsive interaction between the chains of a particular pair ("intrareplica," {12} and {34}, no cross coupling) at the same chain length and is identical to the interaction term used for $\langle Z \rangle$ as discussed in Sec. III and Appendix A. The other term H_2 couples the two pairs of chains {12} and {34} ("interreplica" term) and is the crucial term for our analysis. Even though this is a four-chain interaction, it is distinct from the multicritical-type interaction of Eq. (2.3). It cannot be interpreted directly as a conventional interaction between the two pairs. Rather, there is a lowering of "energy" of the system if the partners of each pair {12} and {34} meet simultaneously at the same chain length but not necessarily at the same point in space. This can also be interpreted as a special correlation so that an encounter of {12} at a chain length z favors an encounter for {34} right at the same length z. A tendency to achieve this kind of configurations leads to all the nontrivial effects of the disorder.

The coupling constant of H_2, $\bar{r}_0$ in Eq. (4.1e), appears to be similar to that of the attractive interaction which is present in H_1, Eqs. (4.1d) and (3.2), but they require separate treatments in the RG analysis (see below). As discussed in Appendix A, the term proportional to $v_0^2 \Delta$ in $\bar{v}_0$ is reduced by a cutoff volume factor Ω needed to define the δ^2 term properly. Because of this reduction $v_0^2 \Delta / \Omega$ differs from $\bar{r}_0$ in dimensionality and matches properly with v_0, the coupling constant of the starting δ function interaction of the two chains.

The standard dimensional analysis for dimensionless Hamiltonian shows that $[v_0] = L^{d-2}$, $[\Delta] = L^2$, and hence $[v_0^2 \Delta] = L^{2d-2}$, where L has the dimension of length. Therefore, the upper critical dimension for $\bar{r}_0$ is $d = 1$ which also appears as a special dimension through the divergences in the ϵ $(= 1 - d)$ expansion to be discussed below. From this simple dimensional analysis it also follows that the coupling in H_2 differs from that of H_1, as already mentioned. The special dimensionality $d = 1$ which is associated with H_2 is more important in the context of fluctuations in the partition function. In the absence of this term there is no special effect of disorder, which, in turn, also means that the quenched and annealed free energies would become equal.

B. Perturbation series

To study the effect of H_2, we develop a perturbation series for $\langle Z^2 \rangle$ in $\bar{r}_0$. The divergences that appear are absorbed by renormalization through an ϵ expansion. We show that this renormalization can be carried out exactly to all orders. For simplicity, this is done first for the $\bar{v}_0 = 0$ case. We then show that these divergences at $d = 1$, arising only due to H_2, remain unchanged even if we include H_1, i.e., when $\bar{v}_0 \neq 0$. The β function evaluated exactly to all orders in perturbation series and other essential features are identical for both $\bar{v}_0 = 0$ and $\bar{v}_0 \neq 0$ cases.

1. $\bar{v}_0 = 0$

Let us consider first, for simplicity, the case when $\bar{v}_0 = 0$. This means that there is no mutual two-chain interaction. We consider only the connected part $\langle Z^2 \rangle_c = \langle Z^2 \rangle - \langle Z \rangle^2$, the second cumulant of the partition function. As in Sec. III, the calculation can be done in the real (chain) space. But, at this point, we prefer the Laplace space (Laplace transform with respect to the chain length) because it is advantageous for later considerations especially with $\bar{v}_0 \neq 0$. We define

$$\mathcal{Z} = \int_0^\infty dN \; e^{-sN} \langle Z^2 \rangle_c, \tag{4.2}$$

the Laplace transform of $\langle Z^2 \rangle_c$, with respect to the chain-length N, the Laplace conjugate variable being s.

The loops in the perturbation expansion are shown in Fig. 2(a) up to third order in the interaction. The individual pairs of chains are represented by thick lines. The horizontal wiggly lines in these diagrams stand for $\bar{r}_0$. Such a representation is possible because the δ function in H_2, Eq. (4.1e), forces the members of a pair to have

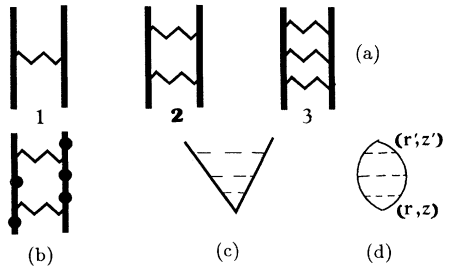

FIG. 2. (a) The only contributing diagrams in $\langle Z^2\rangle\,|_{\bar v_0=0}$ up to third order. Only ladder diagrams occur. A thick line corresponds to a pair of chains. A wiggly line stands for an $\bar r_0$ factor in the evaluation of the diagrams. (b) A typical diagram for $\langle Z^2\rangle\,|_{\bar v_0\neq 0}$. The dots on the thick lines represent intrapair interactions ($\bar v_0$). (c) The dressed propagator with two chains tied at only one end. The dashed lines represent the mutual δ-function-type interaction with coupling constant $\bar v_0$. (d) The dressed propagator with two chains tied at both $(\mathbf{r}, z)$ and $(\mathbf{r}', z')$.

the same $\mathbf{r}, z$ coordinates. Each chain is described by the free distribution ("propagator") $G(\mathbf{r}_f - \mathbf{r}_i \mid z_f - z_i) = [2\pi(z_f - z_i)]^{-d/2}\exp[-(\mathbf{r}_f - \mathbf{r}_i)^2/2(z_f - z_i)]$ with end points $(\mathbf{r}_f, z_f)$ and $(\mathbf{r}_i, z_i)$. Two chains are therefore described by

$$G^2(\mathbf{r} \mid z) = (4\pi z)^{-d/2}\, G(\mathbf{r} \mid z/2). \tag{4.3}$$

This G^2 is the propagator for the thick lines. At each wiggly line, connecting four chains (all four having the same chain length z), there are two integrations over the spatial coordinates of the two separate pairs of chains (thick lines). The loops formed out of the wiggly lines are only responsible for the divergences at $d = 1$.

In order to trace the algebraic origin of the singularity, note that, by the very nature of the interaction, the spatial integrations associated with the two thick lines are independent of each other. Each section of the thick lines, with z_1, z_2 as the end points, in a loop formed with the wiggly lines, contributes $(z_1 - z_2)^{-d/2}$ from the identity in Eq. (4.3). Since the interaction demands the same z for the two thick lines, the z integrals involve $(z_1 - z_2)^{-d}$ type factors whose Laplace transform would contribute $\Gamma(1-d)$ with pole at $d = 1$. The two independent spatial coordinates, which are left out after the successive use of the normalization $\int d\mathbf{r}\; G(\mathbf{r} \mid z) = 1$, lead to a $\mathcal{V}^2$ factor for each diagram. The convolution nature of the z integrals, thanks to the time ordering, leads to a simple product of the individual Laplace transforms of the integrands, resulting in a geometric series for $\mathcal{Z}$. The details of the evaluation of a few diagrams are given in Appendix A.

Defining the dimensionless coupling constant r_0 through an arbitrary length scale L as $r_0 = \bar r_0 L^{2\epsilon}(4\pi)^{-d}$, $\epsilon = 1 - d$, we write the series for $\mathcal{Z}$ to all orders in r_0 as

$$\mathcal{Z}\,|_{\bar v_0=0} = (4\pi)^d\, \mathcal{V}^2\, s^{-2}\, L^{-2\epsilon} \times\left(r_0 + \sum_{n=1}^{\infty} r_0^{n+1}\,(sL^2)^{-n\epsilon}\,\Gamma^n(\epsilon)\right). \tag{4.4}$$

It is clear from the above expression that there is a divergence at $d = 1$ at each order (> 1). This is tackled by renormalization below.

2. $\bar v_0 \neq 0$

In the above analysis, we have taken $\bar v_0 = 0$. We now include $\bar v_0$ and show that the singularity structure around $d = 1$, as in Eq. (4.4), remains unaffected.

In this case there are both intrapair and interpair interactions. The intrapair interactions, i.e., the mutual short-range δ-function interactions among the members of the pairs, are represented by dots on the thick lines. The basic idea of the procedure adopted is to show that the dots can be absorbed by dressing the "propagators." The original propagators (thick lines of the $\bar v_0 = 0$ case) G^2 are modified by the dots, but not trivially. The "dressing" factor depends on whether the chains are open or tied at the ends. One therefore needs two types of dressed propagators, $\bar G_M$ for the thick lines in the loops and G_O for the same in the outer legs not involved in the loops. [See Figs. 2(c) and (d).] Since these involve only two-chain interactions, the singularities are at $d = 2$ of the type $\Gamma(1 - d/2)$ as in Eq. (3.4). These dressed propagators are to be used, as appropriate, in the skeleton diagrams of the $\bar v_0 = 0$ case without the dots [see Fig. 2(a)]. We just quote the forms of these propagators below—the details can be found in Appendix B.

For the one in which the two participating members of a thick line are tied together at both the ends [$(\mathbf{o}, 0)$ and $(\mathbf{r}, z)$] of the line, the form of the dressed propagator with n meetings (dots) is [Fig. 2(d)]

$$\bar G_M^{(n)}(\mathbf{r} \mid z) = (-\bar v_0)^n (4\pi)^{-(n+1)d/2}\frac{\Gamma^{n+1}(\epsilon')}{\Gamma((n+1)\epsilon')} \times z^{(n+1)\epsilon'-1} G(\mathbf{r} \mid z/2). \tag{4.5}$$

There is translational invariance in both $\mathbf{r}$ and z. With $\epsilon' = (2-d)/2$, this form surely reduces to Eq. (4.3) for $n = 0$. Similarly, for the other type in which the two members of a thick line are tied together only at one of the ends, integrations for the open end coordinates need to be done. The resulting dressed propagator for chains of length z with n intermediate dots [Fig. 2(c)] has the following form:

$$G_O^{(n)}(z) = \frac{[-z^{\epsilon'}\bar v_0\Gamma(\epsilon')]^n}{\Gamma(1+n\epsilon')(4\pi)^{nd/2}}. \tag{4.6}$$

For $n = 0$, $G_O^{(n)}(z) = 1$, as it should be, by the normalization of the distribution function $G(\mathbf{r} \mid z)$. Also $G_O^{(n)}(z)$ has no space dependence.

In a diagram of a particular order in $\bar r_0$, the thick lines can have arbitrary order n in $\bar v_0$ (i.e., arbitrary number of dots). All such diagrams differing only in orders of $\bar v_0$ are combined together by summing over n. The full dressed propagators are

$$\bar G_M(\mathbf{r} \mid z) = \sum_n \bar G_M^{(n)}(\mathbf{r} \mid z), \quad G_O(z) = \sum_n G_O^{(n)}(z). \tag{4.7}$$

Unlike the $\bar v_0 = 0$ situation, these two propagators replace the inner and the outer thick lines, respectively.

The subsequent procedure is almost similar to the previous case, including the origin of the $\mathcal{V}^2$ factor and the use of the convolution theorem in the Laplace space. The series for $\mathcal{Z}$ is given by

$$\mathcal{Z}\,|_{\bar{v}_0\neq 0} = \mathcal{V}^2\mathcal{G}_O(s)\bar{r}_0\left[1+\sum_{n=1}^{\infty}[\bar{r}_0\mathcal{G}_M(s)]^n\right]\mathcal{G}_O(s), \quad (4.8)$$

where $\mathcal{G}_O(s) = \mathcal{L}G_O^2(z)$, $\mathcal{G}_M(s) = \mathcal{L}G_M^2(z)$ ($\mathcal{L}$ being the Laplace transform with resprect to z), and $G_M(z) = \int d\mathbf{r}\bar{G}_M(\mathbf{r} \mid z)$. The "same z" requirement of the δ function of H_2 combines the propagators of the thick lines. Hence G_O^2 and G_M^2. The two outer pairs of legs contribute the two $\mathcal{G}_0(s)$ factors. In terms of the dimensionless coupling constant r_0 and $\bar{u}_0$ $[= \bar{v}_0(4\pi)^{-d/2}\Gamma(2\epsilon')s^{-\epsilon'}]$, the above expression becomes

$$\mathcal{Z}\,|_{\bar{v}_0\neq 0} = (4\pi)^d\mathcal{V}^2 s^{-2}L^{-2\epsilon}\mathcal{S}_O(s)\left[r_0+\sum_{n=1}^{\infty}r_0^{n+1}(sL^2)^{-n\epsilon}[\mathcal{S}_M(s)]^n\right]\mathcal{S}_O(s), \quad (4.9)$$

where

$$\mathcal{S}_0(s) = \sum_{\{n\}}\frac{\Gamma[(n_1+n_2)\epsilon'+1]}{\Gamma(1+n_1\epsilon')\Gamma(1+n_2\epsilon')}(-\bar{u}_0)^{n_1+n_2} \quad (4.10)$$

and

$$\mathcal{S}_M(s) = \sum_{\{n\}}\frac{\Gamma[(n_1+n_2)\epsilon'+1-d]}{\Gamma((n_1+1)\epsilon')\Gamma((n_2+1)\epsilon')}(-\bar{u}_0)^{n_1+n_2}. \quad (4.11)$$

Details can be found in Appendix B. The reason it is written in the above form is that $\mathcal{S}_0$ and $\mathcal{S}_M$ start with 1 for $n_1 = n_2 = 0$, to agree with Eq. (4.4). It follows that the leading divergences at $d = 1$ in each order of Eq. (4.9) come from the $n_1 = n_2 = \cdots = 0$ term of Eqs. (4.10) and (4.11).

C. Renormalization and new criticality

The divergences at $\epsilon = 0$ in the series of Eq. (4.9) can be absorbed by the standard renormalization procedure [30]. In general, a renormalization through minimal subtraction would require absorption of the poles in ϵ through

$$r_0 = r(1+a_1r+a_2r^2+\cdots), \quad (4.12)$$

with $a_n = \sum_{p=1}^{n}a_{n,p}\epsilon^{-p}$ and r as the renormalized coupling constant. In such a scheme, $a_{n,p}(p \neq n)$ terms are required to take care of the subleading divergences.

The formal similarity of the leading-pole structure of Eqs. (4.4) and (4.9) with that of Eq. (3.4) enables us to follow Refs. [17,12], yielding $a_p = (-\epsilon)^{-p}$. The geometric series of Eqs. (4.9) and (4.4) guarantees that the removal of the leading poles is sufficient to remove the subleading ones. The presence of the dots [Fig. 2(b)] through $n_1, n_2 \neq 0$ in Eq. (4.9) is felt through the changes in the subleading divergences. This does not pose a problem and can indeed be checked explicitly. Note that $\mathcal{S}_M$ of Eq. (4.11) has an expansion of the form

$$\mathcal{S}_M = \frac{1}{\epsilon}+A_0+\sum_{p=1}A_p\epsilon^p. \quad (4.13)$$

Taking $a_p = (-\epsilon)^{-p}$, as needed to remove the leading poles, one can verify explicitly that all the poles are removed order by order, and the result does not depend on the explicit values of A_0, A_1, etc.

The β function is therefore exact to all orders in perturbation series and is given by

$$\beta(r) \equiv L\frac{\partial r}{\partial L} = 2(\epsilon r + r^2). \quad (4.14)$$

There are two fixed points: (i) $r = 0$ and (ii) $r^* = -\epsilon$. The flows are shown in Fig. 3. The bare coupling constant r_0, which originates from $v_0^2\Delta$, where Δ, the variance of the distribution, is strictly positive, requires a positive r. Therefore, the nontrivial fixed point for $d < 1$ in negative r is unphysical. However, it moves to the physical domain for $d > 1$. See Fig. 3(c).

Exactly at $d = 1, \epsilon = 0$, r grows with length L as

$$r(L) = r(0)\left[1+2r(0)\ln\frac{L_0}{L}\right]^{-1}, \quad (4.15)$$

$r(0)$ being the coupling at length L_0. Hence the disorder is marginally relevant, in agreement with Ref. [24]. For $d > 1$, there exists an unstable nontrivial fixed point at $r = \mid \epsilon \mid$ which separates two distinct regimes of disorder. If we start with a strong enough disorder, on the right-hand side of the fixed point, it increases with the length scale, going beyond the perturbative regime. This is the strong disorder phase. On the other hand, the left-hand side of the fixed point is the weak disorder regime, since r flows to zero (the stable fixed point). The unstable fixed point therefore represents a critical point—an unexpected phase transition induced by the disorder.

One way of achieving the above-mentioned critical behavior is to change the strength of the disorder by controlling the temperature. The "strong disorder" phase ($\langle \ln Z\rangle \neq \ln\langle Z\rangle$) would correspond to the low-

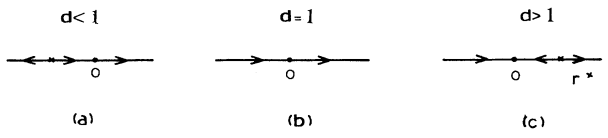

FIG. 3. Flow diagram for r in various dimensions. r^* $(= -\epsilon)$ represents the nontrivial unstable fixed point. For the mth-order multicritical case r is to be replaced by r_m. The three figures would be for $d < d_m$, $d = d_m$, and $d > d_m$, where $d_m = 1/(m-1)$.

temperature phase while the "weak disorder" phase ($\langle \ln Z \rangle \approx \ln \langle Z \rangle$) is the high-temperature one. The details of the critical behavior can be obtained by integrating the β function,

$$r = \mid \epsilon \mid \left[1 - \frac{r(0) - \mid \epsilon \mid}{r(0)} \left(\frac{L}{L_0} \right)^{2 \mid \epsilon \mid} \right]^{-1} . \tag{4.16}$$

For a small starting deviation $\Delta T \equiv T - T_c = r(0) - \mid \epsilon \mid$, there is a length scale $L \sim (\Delta T)^{-1/2\mid\epsilon\mid}$ at which r in Eq. (4.16) diverges. This we can identify as a length scale ξ associated with the critical point with the length scale exponent

$$\nu = (2 \mid \epsilon \mid)^{-1}. \tag{4.17}$$

The divergence at $\epsilon = 0$ is consistent with the essential singularity that follows from Eq. (4.15),

$$\xi \sim \exp[1/(2\Delta T)]. \tag{4.18}$$

A complete description of the critical point would involve an evaluation of various macroscopic or thermodynamic properties. These would require a replica-type analysis. It is tempting to believe that the correlation induced by H_2 in Eq. (4.1e) in the replica space distinguishes the two phases. We wish to come back to such replica analysis elsewhere.

V. $\langle Z^3 \rangle$

The evaluation of $\langle Z^3 \rangle$ leads to a six-chain problem where, as before, an interaction involving four chains is generated which is attractive in nature. The effective Hamiltonian, apart from the free part for six chains and mutual δ function interaction, contains the following attractive terms [see Eq. (4.1b)]:

$$-\bar{r}_0 \int_0^N dz \, [\delta(\mathbf{r}_{12}(z))\, \delta(\mathbf{r}_{34}(z)) + \delta(\mathbf{r}_{34}(z))\, \delta(\mathbf{r}_{56}(z)) + \delta(\mathbf{r}_{12}(z))\, \delta(\mathbf{r}_{56}(z))].$$

Instead of $\langle Z^3 \rangle$, we analyze the third cumulant $\langle Z^3 \rangle_c$ involving only six-chain connected diagrams. Figure 4 shows such diagrams upto fourth order in $\bar{r}_0$. Contributions of these diagrams can be found out following the rules discussed in the context of $\langle Z^2 \rangle$ in Sec. IV. As an example, we give an explicit evaluation of Fig. 4(c), which is

$$\bar{r}_0^4 \int_{\{\mathbf{r},\mathbf{r}',\mathbf{r}''\}} \int_0^N dz_1 \int_0^{z_1} dz_2 \int_0^{z_2} dz_3 \int_0^{z_3} dz_4 \; G^2(\mathbf{r}_{12} \mid z_{12})\; G^2(\mathbf{r}'_{12} \mid z_{12}) G^2(\mathbf{r}'_{23} \mid z_{23}) G^2(\mathbf{r}'_{34} \mid z_{34})\; G^2(\mathbf{r}''_{34} \mid z_{34}).$$

In the above equation $\mathbf{r}, \mathbf{r}', \mathbf{r}''$ with appropriate subscripts denote the set of d-dimensional coordinates for the three thick lines and $\int_{\{\mathbf{r},\mathbf{r}',\mathbf{r}''\}}$ corresponds to the integrations over all spatial coordinates. As before, each thick line between two end points $(\mathbf{r}_i, z_i)$ and $(\mathbf{r}_j, z_j)$ is represented by $G^2(\mathbf{r}_{ij} \mid z_{ij})$ with $z_{ij} = z_i - z_j$. The spatial integrations simplifies the above expression to

$$\bar{r}_0^4 (4\pi)^{-5d/2} \mathcal{V}^3 \int_0^N dz_1 \int_0^{z_1} dz_2 \int_0^{z_2} dz_3 \int_0^{z_3} dz_4 \; z_{12}^{-d}\; z_{23}^{-d/2}\; z_{34}^{-d} = \bar{r}_0^4 (4\pi)^{-5d/2} \mathcal{V}^3 \frac{\Gamma^2(1-d)\; \Gamma(1-d/2)\; N^{4-5d/2}}{\Gamma(5-5d/2)}. \tag{5.1}$$

The series for the Laplace transform of $\langle Z^3 \rangle_c$ is given by

$$[\mathcal{L}\langle Z^3 \rangle_c] \mid_{\bar{v}_0 = 0} = \mathcal{V}^3 (4\pi)^{3d/2} s^{-1-3d/2} \Gamma(\epsilon') [2 {r_0}^2 (sL^2)^{-2\epsilon} + 4 {r_0}^3 (sL^2)^{-3\epsilon} \Gamma(\epsilon) + 6 {r_0}^4 (sL^2)^{-4\epsilon} \Gamma^2(\epsilon) + \cdots] \tag{5.2}$$

where, as before, s is the Laplace conjugate to the chain length N and r_0 is the dimensionless coupling constant as defined before Eq. (4.4). This series requires the standard renormalization procedure for removal of divergence at $d = 1$. Defining the renormalized r via

$$r_0 = r(1 + a_1 r + a_2 r^2 + \cdots) \tag{5.3a}$$

it is found that

$$a_p = (-1/\epsilon)^p \tag{5.3b}$$

absorbs the divergence at $d = 1$.

It is interesting to note that at a particular order there are diagrams which are similar by a mere permutaiton of the interaction lines, i.e., by a different time ordering. For example, Fig. 4(b) shows four diagrams related by permutations, in the third order of the perturbation series. All of these have the same value and hence the factor of 4 in the r_0^3 term in Eq. (5.2). These permutation factors collaborate with powers of r_0 in such a way that the a_p are just the same as those for the $\langle Z^2 \rangle$ case. There are also diagrams in the third and higher orders [a few shown in Fig. 4(d)] which correspond to subleading divergences, the removal of which will be automatic by their corresponding higher orders.

Therefore we see that the β function for r has the identical form to that in Eq. (4.14) for $\langle Z^2 \rangle_c$ and all the features follow identically. This shows that the phase transition for $\langle Z^3 \rangle_c$ has the same nature as for the $\langle Z^2 \rangle$ case. To be more explicit, there exists a transition temperature for $d > 1$ which separates the weak disorder and strong disorder phase for every moment. In the high-temperature phase $\langle Z^3 \rangle \sim \langle Z \rangle^3$ and for $T < T_c$, i.e., in the fluctuation dominated phase, $\langle Z^3 \rangle$ differs from $\langle Z \rangle^3$. This transition temperature is the same for $\langle Z^3 \rangle$ and $\langle Z^2 \rangle$.

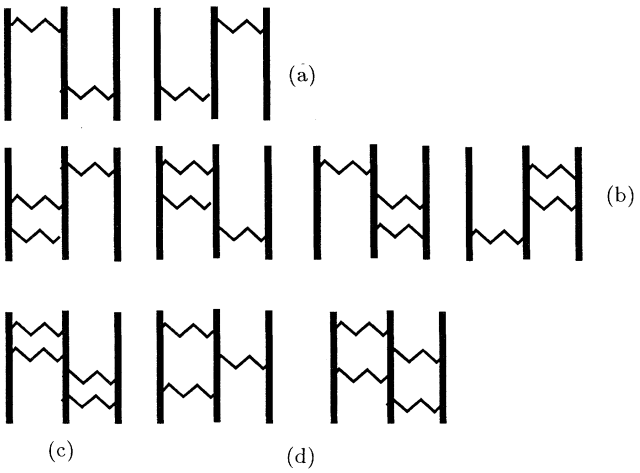

FIG. 4. The (a) second-, (b) third-, and (c) fourth-order (in $\bar{r}_0$) connected diagrams for $\langle Z^3 \rangle \mid_{\bar{v}_0=0}$. In fourth order, there are a few other similar diagrams which contribute to the leading divergence. For connectedness, the series has to start at order 2. (d) Diagrams which contribute to subleading divergences in the third and fourth order in $\bar{r}_0$.

It is now a trivial exercise to extend this for higher moments [31]. The effective Hamiltonian, apart from the free part for $2n$ chains and mutual δ-function interaction, involves the following attractive interaction:

$$-\bar{r}_0 \sum_{\substack{i,j \\ i<j}} \int_0^N dz\ \delta(\mathbf{r}_{2i-1\ 2i}(z))\ \delta(\mathbf{r}_{2j-1\ 2j}(z)).$$

Since no new interaction is generated, the β function remains the same [25, 30].

VI. RANDOM MULTICRITICAL CASE

In the preceding sections, attention was focused on the two-body interaction case. It is known that DP's with pure m-body interaction can also be completely solved [26, 18, 19]. We now investigate the random version of this multicritical case as given by the Hamiltonian of Eq. (2.3). As before, we want to evaluate $\langle Z_m \rangle$ and $\langle Z_m^2 \rangle$. The procedure follows the footsteps of the two-chain problem, therefore details are skipped.

A. $\langle Z_m \rangle$

To compute $\langle Z_m \rangle$, we can perform an averaging over $b(z)$ to obtain, as in Sec. III, an m-chain Hamiltonian with a pure m-body interaction. The grand universality known for the pure system indicates that the multicritical exponents for the binding-unbinding transition will be similar to those of Ref. [29]. For example, for $d > 2/(m-1)$, the length scale exponent would be $2/\mid \epsilon'_m \mid$, where $\epsilon'_m = 2 - (m-1)d$.

B. $\langle Z_m^2 \rangle$

A little calculation involving the completion of the square would convince the reader that the effective Hamiltonian needed for the second moment would involve $2m$ chains in sets of m. It is given by

$$\mathcal{H}_{m,m} = \frac{1}{2} \int_0^N dz \sum_{i=1}^{2m} \left(\frac{\partial \mathbf{r}_i(z)}{\partial z} \right)^2 + \bar{v}_m \int_0^N dz \prod_{p=1}^{m-1} \delta(\mathbf{r}_{p\ p+1}(z)) + \bar{v}_m \int_0^N dz \prod_{q=m+1}^{2m-1} \delta(\mathbf{r}_{q\ q+1}(z))$$
$$-\bar{r}_m \int_0^N dz \prod_{p=1}^{m-1} \delta(\mathbf{r}_{p\ p+1}(z)) \prod_{q=m+1}^{2m-1} \delta(\mathbf{r}_{q\ q+1}(z)), \tag{6.1}$$

where $\bar{r}_m = v_m^2 \Delta$ and the two sets are represented by p's and q's. The special feature is the last term that involves the peculiar m-chain–m-chain interaction. This generalizes H_2 of the two-chain case of Eqs. (4.1b) and (4.1e). The effect of disorder, as far as the fluctuations are concerned, is to introduce a correlation that if m chains meet at a z, the replica would also like to meet at that same z.

The upper critical dimension of r_m follows from the dimensional analysis as $d_m = 1/(m-1)$, which is half of the upper critical dimension for the pure case $[2/(m-1)$ for $\bar{v}_m]$. We are not sure whether this systematic reduction by a factor of 2 has any deeper significance.

For simplicity we choose $\bar{v}_m = 0$. The perturbation expansion in $\bar{r}_m$ would involve the same sets of diagrams as in Fig. 2(a) except that now the propagator for the thick lines is $G^m(\mathbf{r} \mid z) = (2\pi z)^{-(m-1)d/2} m^{-d/2} G(\mathbf{r} \mid z/m)$. With this propagator, the full series can be computed. It is transparent to see the occurrence of divergences at $d = d_m$. The whole RG procedure of Sec. IV can be copied in toto by replacing ϵ by $\epsilon_m = 1 - d(m-1)$. Hence in the multicritical situation we also expect to see a disorder-induced phase transition. The length scale exponent $\nu_m = (2 \mid \epsilon_m \mid)^{-1}$, with an essential singularity for $d = d_m$ as in Eq. (4.18).

VII. SYSTEM WITH MORE THAN TWO CHAINS

Annealed averaging for the system with two chains with random interaction is simpler and not sufficient to give enough information about the effects of disorder. On the other hand, if the above case can be extended to four chains having two-body interaction among each other, even the annealed case turns out to be extremely nontrivial. The Hamiltonian for the four-chain system

$$\mathsf{H} = \frac{1}{2}\int_0^N dz \sum_{i=1}^{4}\left(\frac{\partial \mathbf{r}_i(z)}{\partial z}\right)^2 + \int_0^N dz\, v_0[1+b(z)] \sum_{\substack{i,j=1\\ i\neq j}}^{4} \delta(\mathbf{r}_{ij}(z)) \tag{7.1}$$

where $\mathbf{r}_{ij} = \mathbf{r}_i(z) - \mathbf{r}_j(z)$, after averaging, using the Gaussian distribution of $b(z)$, gives the following effective Hamiltonian:

$$\mathcal{H}_{\text{eff}} = \frac{1}{2}\int_0^N dz \sum_{i=1}^{4}\left(\frac{\partial \mathbf{r}_i(z)}{\partial z}\right)^2 + \left(\frac{\partial \mathbf{r}_i(z)}{\partial z}\right)^2 + \bar{v}_0 \int_0^N dz \sum_{i,j=1}^{4} \delta(\mathbf{r}_{ij}(z)) -$$
$$-2v_0^2\Delta \int_0^N \sum_{i,j,k} \delta(\mathbf{r}_{ij})\delta(\mathbf{r}_{jk}) - 2v_0^2\Delta \int_0^N dz \sum_{i\neq j\neq k\neq l} \delta(\mathbf{r}_{ij})\delta(\mathbf{r}_{kl}). \tag{7.2}$$

The remarkable feature of the effective Hamiltonian is that there are two different kinds of attractive interaction, one of which involves three chains with a multicritical-type interaction [Eq. (2.3)] while the other one couples four chains together, as in the quenched problem.

If we take a three-chain system, the corresponding effective Hamiltonian will involve only the three-chain term but no four-chain interaction of Eq. (7.2). This term was absent in the original three-chain Hamiltonian. There is now the possibility of a disorder-induced multicritical behavior, though of pure type [26, 18].

The four-chain attractive interaction is marginal at $d = 1$ and so is the three-chain interaction. The presence of these two marginal operators is, in general, expected to complicate the renormalization procedure through their interdependence, but here that does not happen.

The perturbation expansion with the three-body and the four-body interactions leads to three different kinds of diagrams. See Fig. 5. The series corresponding to the pure three-body interaction is already solved [26, 18]. The series in the Laplace space involving four-body interactions [see Figs. 5(a)–5(c)], which contributes to the leading divergence, is identical to the series for $\langle Z^2\rangle_c$ in Eq. (4.4) [(Fig. 2(a)]. The diagrams with mixed three-body and four-body interactions are shown in Figs. 5(d) and 5(e). In the final series, up to the order shown in Fig. 5, the four-body and three-body contributions get separated into two factors. This shows that the resulting renormalization of the two couplings are independent of each other. Because of the four-body interaction, we expect a disorder-induced criticality as for the two-chain quenched case, but here this happens for a real four-chain system—no replica is involved. The details and the phase diagram will be published elsewhere.

VIII. SUMMARY AND DISCUSSION

We have proposed a random-interaction model for two directed polymers and studied the first three cumulants of the partition function. We have shown that in the annealed case, described by $\langle Z\rangle$, there can be a disorder-induced binding-unbinding transition, very similar to a pure problem. The exponents are also identical to the pure case. We also pointed out certain peculiarities of the annealed problem involving three or four chains. The quenched problem is different, as reflected through the marginal relevance of the disorder. For $d > 1$ there exists a critical point that demarcates a disorder-dominated phase and a pure-type phase. In the strong disorder phase, there seems to have an extra correlation in the replica space which is absent in the other phase. The length scale exponent for the critical point is found to be $(2 \mid \epsilon \mid)^{-1}$, where $\epsilon = 1 - d$. It has an exponential divergence at $d = 1$. Similar results were obtained for $\langle Z^3\rangle$. In the replica approach, one needs $\langle Z^n\rangle$ with $n \to 0$, which does not require interactions other than those which are present in $\langle Z^2\rangle$ and $\langle Z^3\rangle$. Therefore the upper critical dimension will remain the same, namely $d = 1$.

There are still many open problems, such as, for example, a replica analysis for this system. This requires an explicit expression for $\langle Z^n\rangle$, correct at least for small n. Such an analysis would provide vital information regarding the disorder-induced critical point, including possible replica symmetry breaking. Is there any other length scale exponent for this new crtical point, apart from the one we have calculated? What about other exponents?

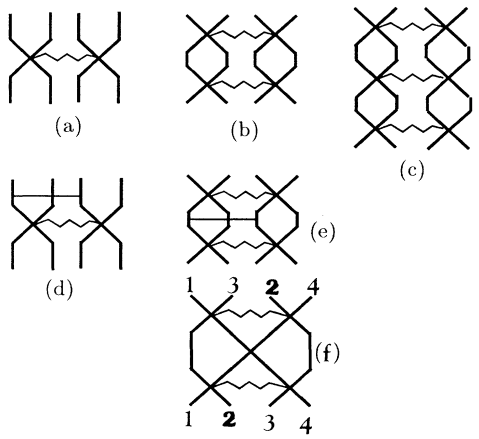

FIG. 5. Four chain diagrams for the annealed problem with four chains. The wiggly lines represent $\bar{r}_0$-type interaction and a solid horizontal line connecting three-chains is the three-chain δ-function interaction. The first three terms of the series involving only four chain interactions are shown in (a), (b), and (c). (d) and (e) Two cases involving both the three- and four-chain interactions. (f) A possible diagram in second order with different chain combinations for the interactions. This contributes in the subleading divergence.

What, if any, is the upper critical dimension of this critical point? A thorough numerical study of this system will surely provide valuable insights.

APPENDIX A: GENERATION OF ATTRACTION: PERTURBATION ANALYSIS FOR $\langle Z \rangle$ AND $\langle Z^2 \rangle$

In this appendix, we show how the attractive terms in the effective Hamiltonian for $\langle Z \rangle$ and $\langle Z^2 \rangle$ can be generated perturbatively.

We proceed to the evaluation of the average of the partition function $\langle Z \rangle$ by a perturbation expansion using Eq. (2.1) with the replacement of the short-range potential by a δ-function potential. Formally this leads to the expression for

$$Z = \int Dr_1 Dr_2 \exp(-H_0)\ (1 - H_i + {H_i}^2/2! + \cdots), \tag{A1}$$

where H_0 corresponds to the free part of the two chains and H_i represents the interaction part $\int_0^N dz\ v_0\ [1 + b(z)]\delta(\mathbf{r}_{12}(z))$. A little manipulation after substituting the explicit form of H_i gives the connected partition function as

$$\begin{aligned} Z_c = &\int_0^N dz_1\ v_0\ [1 + b(z_1)] \int_{\{r,r'\}} G(\mathbf{r}_N - \mathbf{r}_1 \mid N - z_1)\ G(\mathbf{r}_1 - \mathbf{r}_0 \mid z_1) G(\mathbf{r'}_N - \mathbf{r}_1 \mid N - z_1)\ G(\mathbf{r}_1 - \mathbf{r'}_0 \mid z_1) \\ &+ \int_0^N dz_1 \int_0^{z_1} dz_2\ v_0^2\ [1 + b(z_1)][1 + b(z_2)] \int_{\{r,r'\}} G(\mathbf{r}_N - \mathbf{r}_1 \mid N - z_1)\ G(\mathbf{r}_{12} \mid z_{12})\ G(\mathbf{r}_2 - \mathbf{r}_0 \mid z_2) \\ &\times G(\mathbf{r'}_N - \mathbf{r}_1 \mid N - z_1) G(\mathbf{r}_{12} \mid z_{12}) G(\mathbf{r}_2 - \mathbf{r'}_0 \mid z_2) + \cdots, \end{aligned} \tag{A2}$$

where $G(\mathbf{r} \mid z) = (2\pi z)^{(-d/2)} \exp(-r^2/2z)$ is the distribution function for a d-dimensional Gaussian chain of chain length z and end to end distance vector $\mathbf{r}$, and $\int_{\{r,r'\}}$ signifies integrations over all dummy spatial coordinates. The convention is to use $\mathbf{r}$ and $\mathbf{r}'$ for the two chains and $z_{ij} = z_i - z_j$. The factorials in the denominators of the terms of Eq. (A1) are absorbed by introducing "time" (z) ordering, which restricts $z_{i+1} \leq z_i$, in the integrals. Diagrams up to second order corresponding to the series in Eq. (A2) are shown in Fig. 6. The familiar normalization $\int d\mathbf{r}\, G(r \mid z) = 1$ and the integrations over the spatial end coordinates lead to a simplification for Z_c. The explicit form of Z_c up to two loop term is given by

$$\begin{aligned} Z_c = &\mathcal{V} \int_0^N dz_1 v_0 [1 + b(z_1)] - \int_0^N dz_1 \int_0^{z_1} dz_2 \int_{\{r\}} v_0^2\ [1 + b(z_1)][1 + b(z_2)] \\ &\times G^2(\mathbf{r}_{12} \mid z_{12}) + \int_0^N dz_1 \int_0^{z_1} dz_2 \int_0^{z_2} dz_3 \int_{\{r\}} v_0^3 [1 + b(z_1)][1 + b(z_2)] \\ &\times [1 + b(z_3)] G^2(\mathbf{r}_{12} \mid z_{12}) G^2(\mathbf{r}_{23} \mid z_{23}) \cdots, \end{aligned} \tag{A3}$$

where $\mathcal{V}$ is the transverse volume. The actual meaningful quantity $\langle Z_c \rangle$ is computed from Eq. (A3) after averaging it with the distribution $P(b)$ of Eq. (2.2a). This Gaussian distribution with zero mean ensures that any term involving an odd number of $b(z)$'s should vanish after averaging. Therefore, the contribution from the first-order term $v_0 N \mathcal{V}$ is only from the pure part. In the one-loop level of Eq. (A3), there are two surviving terms after disorder averaging. One is the pure term, which does not require any averaging, and its contribution to $\langle Z_c \rangle$ is

$$-v_0^2 \mathcal{V} (4\pi)^{-d/2} \frac{\Gamma(1 - d/2)}{\Gamma(3 - d/2)} N^{2-d/2}. \tag{A4}$$

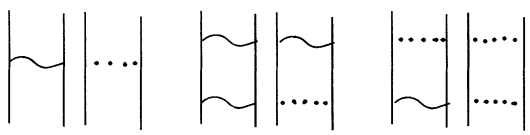

FIG. 6. Diagrams for $\langle Z \rangle$ up to second order in v_0 and $v_0 b(z)$. The wavy and dotted lines represent interactions with coupling constants $v_0 b(z)$ and v_0, respectively.

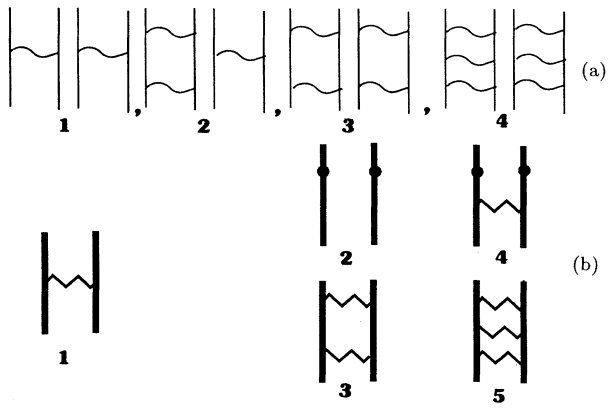

FIG. 7. (a) Diagrams involving only $v_0 b(z)$ for $\langle Z^2 \rangle$. (b) Diagrams of (a) after disorder averaging (see the caption of Fig 2). (a2), With an odd number of wavy lines, vanishes after DA. Different pairings lead to two possiblities (b2) and (b3) from (a3). Similarly for (a4), there are two diagrams (b4) and (b5) after DA. Diagrams (b2) and (b4) are not considered for the $\langle Z^2 \rangle\,|_{v_0=0}$ case.

The other nonvanishing part which contains an even number of disorder interaction is

$$-v_0^2 \int_0^N dz_1 \int_0^{z_1} dz_2 \int d\mathbf{r}_1\, d\mathbf{r}_2\, b(z_1)\, b(z_2)\, G^2(\mathbf{r}_{12} \mid z_{12}).$$

After averaging, the two points z_1 and z_2 along the chain merge together giving rise to a term

$$-\int_0^N dz_1\, v_0^2\, \Delta\delta^2(\mathbf{r}_1 - \mathbf{r}_2)\, d\mathbf{r}_1\, d\mathbf{r}_2, \tag{A5}$$

where we used the fact that $G(\mathbf{r} \mid 0) = \delta(\mathbf{r})$. Because of this merging of the two points along the chain, this term contributes to the first-order term but with a negative sign, which shows the presence of a newly generated attraction. In other words, a second-order term for a particular realization (before averaging) looks like an attractive first-order term after disorder averaging (DA).

Proceeding in the same fashion we can evaluate the two-loop term of Eq. (A3). In the two-loop part, the nonvanishing contributions are

(i) $$v_0^3 \int_0^N dz_1 \int_0^{z_1} dz_2 \int_0^{z_2} dz_3 \int_{\{r\}} G^2(\mathbf{r}_{12} \mid z_{12}) G^2(\mathbf{r}_{23} \mid z_{23}).$$

This term, involving only pure-type interaction, after integration over spatial coordinates, gives

$$\mathcal{V} v_0^3 (4\pi)^{-d} \int_0^N dz_1 \int_0^{z_1} dz_2 \int_0^{z_2} dz_3\, (z_1 - z_2)^{-d/2} (z_2 - z_3)^{-d/2} = \mathcal{V} v_0^3 (4\pi)^{-d} \frac{\Gamma^2(1 - d/2)}{\Gamma(4 - d/2)} N^{3-d/2}. \tag{A6}$$

(ii) $$v_0^3 \int_0^N dz_1 \int_0^{z_1} dz_2 \int_0^{z_2} dz_3 \int_{\{r\}} b(z_1) b(z_2)\, G^2(\mathbf{r}_{12} \mid z_{12}) G^2(\mathbf{r}_{23} \mid z_{23}).$$

As in the earlier case this reduces to a one-loop term after averaging as

$$v_0^3 \Delta\, (4\pi)^{-d/2} \int_0^N dz_1 \int_0^{z_1} dz_2 \int_0^{z_2} dz_3 \int_{\{r\}} \delta^2(\mathbf{r}_{12})\, (z_2 - z_3)^{-d/2}. \tag{A7}$$

(iii) The third nonvanishing contribution is from

$$v_0^3 \int_0^N dz_1 \int_0^{z_1} dz_2 \int_0^{z_2} dz_3 \int_{\{r\}} b(z_2)\, b(z_3)\, G^2(\mathbf{r}_{12} \mid z_{12})\, G^2(\mathbf{r}_{23} \mid z_{23}). \tag{A8}$$

This term after averaging becomes

$$v_0^3 \Delta\, (4\pi)^{-d/2} \int_0^N dz_1 \int_0^{z_1} dz_2 \int_{\{r\}} \delta^2(\mathbf{r}_{23})\, (z_1 - z_2)^{-d/2}. \tag{A9}$$

There is one more term involving $b(z_1)b(z_3)$ which vanishes after averaging because of the specific time ordering which rules out the merging of z_1 and z_3. It can be easily be checked that this merging of two "random interaction" lines (wavy lines) into one single pure line and the subsequent reduction of order occurs at each order (> 1) involving consecutive pairs of even number of "random" lines. Thus an attraction is generated at each order very systematically. The coupling constant of this term is proportional to $v_0^2\Delta$. In some of the above expressions [(A5), (A7), and (A9)] the presence of the δ^2 term needs special attention since it is ill defined even in the sense of the theory of distribution [32]. We can avoid this problem by taking a spread out δ function and then taking the limit at the end. This would change the coupling to $v_0^2\Delta/\Omega$, where Ω is the arbitrary "spread out" or cutoff volume. Since one gets back a single δ function, it can be associated with the pure term, thereby changing the problem to a pure one with a reduced coupling constant $\bar{v}_0 = v_0 - v_0^2\Delta/\Omega$. See Eq. (3.2). Another way to tackle this difficulty is to start with a short-range potential $V(\mathbf{r})$ and appeal to RG arguments as done in Sec. III.

Now we discuss $\langle Z^2 \rangle$. A few diagrams for $\langle Z^2 \rangle$ with $\bar{v}_0 = 0$ are shown in Fig. 7(a). In first order, the only diagram which has nonzero contribution to $\langle Z^2 \rangle_c$ is Fig. 7(a)1. This contribution after disorder averaging is $\mathcal{V}^2 v_0^2 \Delta N$. As was mentioned in the text this $\mathcal{V}^2$ has come from the independent spatial integrals. Proceeding in the similar fashion, we write

$$\text{Fig. 7(a)3} = v_0^4 \int_0^N dz_1 \int_0^{z_1} dz_2\, b(z_1) b(z_2) \int_{\{r\}} G^2(\mathbf{r}_{12} \mid z_{12}) \int_0^N dz_1' \int_0^{z_1'} dz_2' b(z_1') b(z_2') \int_{\{r,r'\}} G^2(\mathbf{r'}_{12} \mid z'_{12}).$$

In disorder averaging, the only relevant contribution comes from the pairing of $b(z_1)$,$b(z_1')$ and $b(z_2)$,$b(z_2')$. The other possibility in which $b(z_1)$,$b(z_2)$ and $b(z_1')$,$b(z_2')$ are paired up [Fig. 7(b)2] is not considered here since this generates $\bar{v}_0$-type terms which are not to be included for the $\bar{v}_0 = 0$ case. After appropriate disorder averaging the above expression becomes

$$v_0^4\Delta^2\mathcal{V}^2 4\pi^d\Gamma(\epsilon)s^{-(2+\epsilon)}, \tag{A10}$$

where $\epsilon = 1 - d$.

To make the evaluation, after DA, easier we follow a different convention for the diagrams, Figs. 7(b) and 2(a). The thick line represents the two members of a pair jointly and is represented by $G^2(\mathbf{r}_1 - \mathbf{r}_2 \mid z_1 - z_2)$ for the ends $\mathbf{r}_1$ and $\mathbf{r}_2$ at which the two chains are tied at lengths z_1 and z_2. For example, the diagram which corresponds to the last expression [Eq. (A10)] is given by Fig. 7(b)3, which is also Fig. 2(a)2. (Note that $\bar{r}_0 = v_0^2\Delta$.)

The next diagram which is important in the next higher order is given in Fig. 7(a)4, the contribution of which can equivalently be calculated from Fig. 7(b)5 or 2(a)3 as

$$(v_0^2\Delta)^3\int_0^N dz_1\int_0^{z_1} dz_2\int_0^{z_2} dz_3\int_{\{r,\,r'\}} G^2(\mathbf{r}_{12}\mid z_{12})G^2(\mathbf{r}_{23}\mid z_{23})G^2(\mathbf{r'}_{12}\mid z_{12})G^2(\mathbf{r'}_{23}\mid z_{23}). \tag{A11}$$

In the Laplace space this becomes

$$(v_0^2\Delta)^3\mathcal{V}^2\Gamma^2(\epsilon)(4\pi)^{-2d}s^{-(2+2\epsilon)}. \tag{A12}$$

The diagrams having an odd number of wiggly lines trivially vanishes after DA. This can be generalized to arbitrary orders since only ladder-type diagrams are involved. Equation (4.4) would follow by substitution $\bar{r}_0 = v_0^2\Delta$.

APPENDIX B: DRESSED PROPAGATORS AND $\langle Z^2\rangle$

We first show the two different dressed propagators. The one for which both the chains, tied at the ends $(\mathbf{r}, \mathbf{z})$ and $(\mathbf{r}', z')$, meet each other n times at $(\mathbf{r}_1, z_1)$, $(\mathbf{r}_2, z_2)$, $\cdots(\mathbf{r}_n, z_n)$ [Fig. 2(d)] is given by

$$\bar{G}_M^{(n)}(\mathbf{r}-\mathbf{r}'\mid z-z') = \bar{v}_0^n\int_{\{\mathbf{r}\}}\int_{z'}^{z} dz_1\int_{z'}^{z_1} dz_2\cdots\int_{z'}^{z_{n-1}} dz_n G^2(\mathbf{r}-\mathbf{r}_1\mid z-z_1)$$
$$\times G^2(\mathbf{r}_{12}\mid z_{12})\cdots G^2(\mathbf{r}_{n-1n}\mid z_{n-1n})G^2(\mathbf{r}_n-\mathbf{r}'\mid z_n-z').$$

Use of the identity $G^2(\mathbf{r}\mid z) = (4\pi z)^{-d/2}G(\mathbf{r}\mid z/2)$ and the Markovian property

$$\int d\mathbf{r}_2 G(\mathbf{r}_1-\mathbf{r}_2\mid z_1)G(\mathbf{r}_2-\mathbf{r}_3\mid z_2) = G(\mathbf{r}_1-\mathbf{r}_3\mid z_1+z_2)$$

leads to the following expression for $\bar{G}_M^{(n)}(r-r'\mid z-z')$:

$$\bar{G}_M^{(n)}(\mathbf{r}-\mathbf{r}'\mid z-z') = \bar{v}_0^n 4\pi^{-nd/2}G(\mathbf{r}-\mathbf{r}'\mid (z-z')/2)\int_{z'}^{z} dz_1\int_{z'}^{z_1} dz_2\cdots\int_{z'}^{z_{n-1}} dz_n (z-z_1)^{-d/2}z_{12}^{-d/2}\cdots$$
$$\times z_{n-1n}^{-d/2}(z_n-z')^{-d/2}.$$

So we need the following integral:

$$\int_{z'}^{z} dz_1(z-z_1)^{-d/2}\int_{z'}^{z_1} dz_2(z_1-z_2)^{-d/2}\cdots\int_{z'}^{z_{n-1}} dz_n(z_{n-1}-z_n)^{-d/2}(z_n-z')^{-d/2}.$$

A change of variable

$$\bar{z}_i = z_i - z'$$

and use of the convolution theorem in the Laplace space straightaway yields

$$s^{-\epsilon'(n+1)}\Gamma^{n+1}(\epsilon'),$$

where s is the Laplace conjugate to the chain length (more precisely to $\bar{z} = z - z'$). Converting this to inverse Laplace space and combining all other factors, the final form of such a propagator becomes

$$\bar{G}_M^{(n)}(\mathbf{r}\mid z) = (-\bar{v}_0)^n(4\pi)^{-(n+1)d/2}z^{(n+1)\epsilon'-1}$$
$$\times\frac{\Gamma^{n+1}(\epsilon')}{\Gamma((n+1)\epsilon')}\,G(\mathbf{r}\mid z/2). \tag{B1}$$

Following the same track, the other dressed propagator, for which the two member chains are tied only at one end say $(\mathbf{r}, z)$ other than n meetings, has the form [see Fig. 2(c)]

$$G_O^{(n)}(z) = (-\bar{v}_0)^n(4\pi)^{-nd/2}(z)^{n\epsilon'}\frac{\Gamma^n(\epsilon')}{\Gamma(n\epsilon'+1)}. \tag{B2}$$

This propagator is independent of any space coordinate because of the spatial integration over the open end coordinates.

To take care of arbitrary number of meetings we sum over n. Hence the final dressed propagators are

$$G_O(z) = \sum_{n=0}^{\infty} G_O^{(n)}(z),$$

$$\bar{G}_M(\mathbf{r}-\mathbf{r}'\mid z-z') = \sum_{n=0}^{\infty}\bar{G}_M^{(n)}(\mathbf{r}-\mathbf{r}'\mid z-z').$$

Therefore the series for $\langle Z^2\rangle_c$ can be written as

$$\langle Z^2\rangle_c = \bar{r}_0 \int_0^N dz_1 G_O^2(N-z_1)\; G_O^2(z_1) \tag{B3}$$

$$+\bar{r}_0^2 \int_0^N dz_1 \int_0^{z_1} dz_2 \int_{\{r,\, r'\}} G_O^2(N-z_1)\; \bar{G}_M(\mathbf{r}_{12} \mid z_{12})\; \bar{G}_M(\mathbf{r}'_{12} \mid z_{12})\; G_O^2(z_2) + \cdots . \tag{B4}$$

In fact, it is possible to write the whole series to all orders in perturbation. Completing the integrations over the spatial coordinates we get

$$\begin{aligned}\langle Z^2\rangle_c = {} & \mathcal{V}^2\bar{r}_0 \int_0^N dz_1 G_O^2(N-z_1) G_O^2(z_1) + \bar{r}_0^2\mathcal{V}^2 \int_0^N dz_1 \int_0^{z_1} dz_2 G_O^2(N-z_1) \\ & \times G_M^2(z_{12}) G_O^2(z_2) + \bar{r}_0^3\mathcal{V}^2 \int_0^N dz_1 \int_0^{z_1} dz_2 \int_0^{z_2} dz_3\; G_O^2(N-z_1) \\ & \times G_M^2(z_{12}) G_M^2(z_{23}) G_O^2(z_3).\end{aligned} \tag{B5}$$

Here $G_M(z)$ is obtained from $\bar{G}_M(\mathbf{r} \mid z)$ after integration over the spatial coordinate. We use the convolution theorem for Laplace transforms which leads to the following expression for $\mathcal{Z}$ the Laplace transform of $\langle Z^2\rangle_c$ [Eq. (4.2)]:

$$\mathcal{Z}\mid_{\bar{v}_0\neq 0} = \mathcal{V}^2[\bar{r}_0\mathcal{G}_O^2(s) + \bar{r}_0^2\mathcal{G}_O(s)\mathcal{G}_M(s)\mathcal{G}_O(s) + \bar{r}_0^3\mathcal{G}_O(s)\mathcal{G}_M^2(s)\mathcal{G}_O(s) + \cdots] \tag{B6}$$

where $\mathcal{Z} = \int_0^\infty e^{-sN}\langle Z^2\rangle_c$, $\mathcal{G}_p = \int_0^\infty e^{-sN} G_p^2(z)$, with p being O or M. The Laplace transforms $\mathcal{G}_O(s)$ and $\mathcal{G}_M(s)$ are given by

$$\mathcal{G}_O(s) = \sum_{n_1,n_2} \frac{(4\pi)^{-(n_1+n_2)d/2}\bar{v}_0^{n_1+n_2}\Gamma^{n_1+n_2}(\epsilon')\Gamma((n_1+n_2)\epsilon'+1)}{\Gamma(1+n_1\epsilon')\Gamma(1+n_2\epsilon')\; s^{(n_1+n_2)(1-d/2)+1}} \tag{B7}$$

and

$$\mathcal{G}_M(s) = \sum_{n_1,n_2} \frac{(4\pi)^{-(n_1+n_2+2)d/2}\bar{v}_0^{n_1+n_2}\Gamma^{n_1+n_2}(\epsilon')\Gamma((n_1+n_2)\epsilon'+\epsilon)}{\Gamma((1+n_1)\epsilon')\Gamma((1+n_2)\epsilon')\; s^{(n_1+n_2)\epsilon'+\epsilon}}. \tag{B8}$$

Substituting these expressions in Eq. (B6), we get back Eq. (4.9) for $\mathcal{Z}\mid_{\bar{v}_0\neq 0}$ in the Laplace space. The results for $\bar{v}_0 = 0$ can be obtained from the first term of each sum, i.e., for $n_1 = n_2 = 0$.

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